

SCALING RELATIONS FOR PULSED POWER-DRIVEN HYDRODYNAMICS EXPERIMENTS

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Abstract

Pulsed power hydrodynamics experiments typically involve multi-megajoule capacitor banks, imploding thin-walled cylinders of solid-density metal by the electromagnetic force associated with axial currents in excess of ten megamperes. Much of this activity operates with technology developed for other applications (e.g., soft X-ray generation), which can be significantly larger than actually required for the scientific objectives of such experiments. This is particularly true, if imaging diagnostics (e.g., proton radiography) with greater spatial and temporal resolution become available, permitting implosion experiments of smaller scale size. The present paper develops scaling relationships for electromagnetically-driven implosions and indicates the substantial reduction in system energy with reduced scale size.

I. INTRODUCTION

Pulsed power techniques provide excellent opportunities for investigating matter under extreme conditions. One category of technique uses multi-megampere currents to implode solid-density, hollow cylinders (*liners*) in the manner of a z-pinch for which axial current interacts with its own magnetic field to create a radial implosion force. Such implosions readily provide speeds of several km/s, and are capable of creating strong shocks and damage in converging geometries that provide stresses in both radial and azimuthal directions.

The precision of electromagnetic implosion can be better than 1% in space and time, permitting careful diagnosis of physical phenomena and close comparison with computer calculations.

A principal difficulty in recent years has been supporting facilities to provide the necessary multi-megampere currents for liner implosion. In addition to the cost and size of the pulsed power facility, the consequences of the liner implosion itself favor lower energy events in terms of reduced collateral damage, smaller replacement parts and easier connection to diagnostic systems. Recent advances in proton radiography as a high resolution diagnostic technique and continued interest in experiments with environmentally-challenging materials separately call for the development of pulsed power and liner implosion systems that can be brought to special locations (e.g., proton beam-line). We explore here the necessary scaling relations for reducing the system size while retaining important intensive values, e.g., liner speed, compression ratio and energy density.

II. BASIC FORMULATION

To provide true scaling between pulsed power-driven liner implosions as we reduce the system size, we should attempt to achieve the same implosion trajectory, speed and thermodynamic state. It is therefore useful to normalize the equations for the liner and driver-circuit behavior in terms of characteristic values and dimensionless parameters. We start with a simple LRC-circuit driving an imploding-liner load of increasing inductance and resistance as depicted in Fig. 1.

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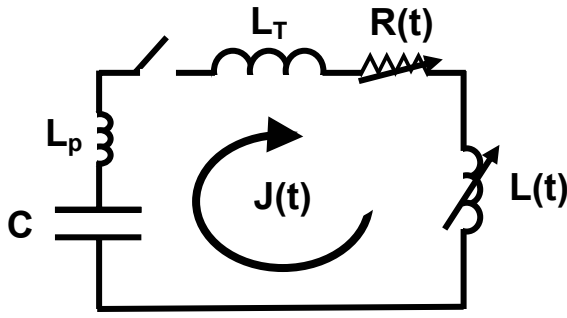


Figure 1. Schematic of simple LRC-circuit driving liner.

The equations for this system in dimensional form are:

$$[L_T + L_p + L(t)] \frac{dJ}{dt} = V - J \left(\frac{dL}{dt} + R \right) \quad (1a)$$

$$\frac{dV}{dt} = -J/C \quad (1b)$$

$$\frac{du}{dt} = (J^2/2m) \frac{dL}{dr} \quad (1c)$$

$$\frac{dr}{dt} = -u \quad (1d)$$

$$\frac{dL}{dt} = u \frac{dL}{dr} \quad (1e)$$

$$\frac{dQ}{dt} = \eta_o (J/A)^2 [1 + \beta Q] \quad (1f)$$

$$R = R_o [1 + \beta Q] \quad (1g)$$

With reasonable transparency, we denote the liner mass by m , inductances by L , resistance by R and capacitance by C . Thus, L_T and L_p are the inductances of the transmission plates and parasitic inductance of the capacitor and switch, respectively; the total initial inductance is $L_o = L_T + L_p$. The resistance displayed in Eqn. 1g is only that of the liner itself, but will be augmented later by additional (constant) series resistance of the plates, switches and connections. The resistivity, η , and resistance, R , of the liner increase linearly (from initial values denoted by the subscript 'o') with the energy per unit volume, Q , deposited by resistive heating. This is adequate for the liner remaining in the solid-state, which is consistent with our neglect of many complex processes captured more appropriately in a fully-posed MHD computer code. In deference to those who use 'I' for *impulse*, we denote the total current by J .

To focus quickly on issues of liner size, we have simplified the formulation to a 'thin-liner approximation' in which the current density is uniform in the liner thickness. The thickness increases during implosion to conserve the cross-sectional area, A , in the manner of an incompressible fluid, but we retain a uniform current-density approximation nevertheless. This approximation permits a first-order estimate of the liner heating in the solid-state and may be checked *a posteriori* to verify that we have not melted the liner completely. Indeed, we should expect that high magnetic field values on the outer surface of the liner will result in melting there. Prior to that, however, we would have more than doubled the electrical resistivity of the material, which corresponds to nonlinear magnetic diffusion, tending to justify the uniform current-density approximation and spreading the heat load through the liner thickness.

Normalization of Eqns. 1 follows by introduction of characteristic values for all the variables. In some cases, the choice of a characteristic value is evident *a priori*, such as use of the initial value of voltage, V_o , on the capacitor to scale subsequent voltages, and the initial liner radius, r_o , to measure the radius further in. Reference to a simple LC-circuit suggests also that the characteristic values for time and impedance are $(L_o C)^{1/2}$ and $(L_o/C)^{1/2}$, respectively. These choices imply a characteristic values for current of $V_o/(L_o/C)^{1/2}$ and for inductance, L_o . We then have dimensionless variables:

$$\tau = t/(L_o C)^{1/2}, \quad v = V/V_o, \quad j = J/[V_o/(L_o/C)^{1/2}]$$

$$\lambda = L(t)/L_o, \quad \theta = R_o/(L_o/C)^{1/2}, \quad q = [1 + \beta Q] \quad (2)$$

$$\omega = u/u_c, \quad \rho = r/r_o$$

where u_c is a characteristic speed not yet specified. Substitution of these variables into Eqns. 1 provides dimensionless differential equations:

$$dj/d\tau = [v - j(\omega/\rho + \theta_c + \theta q)] / (1 + \lambda) \quad (3a)$$

$$dv/d\tau = -j, \quad d\lambda/d\tau = \omega/\rho \quad (3b,c)$$

$$d\omega/d\tau = \Gamma j^2/\rho, \quad d\rho/d\tau = -\Lambda\omega \quad (3d,e)$$

$$dq/d\tau = Kqj^2 \quad (3f)$$

In this process, we find that a useful value of characteristic speed is:

$$u_c = L_{do}/(L_o/C)^{1/2} \quad (4)$$

where L_{do} is the initial value of the inductance gradient dL/dr . We also define five dimensionless parameters:

$$\begin{aligned} &\text{Dynamic parameter} \\ \Gamma &= [CV_o^2/2m]/[(L_o/C)^{1/2}/L_{do}]^2 \end{aligned} \quad (5a)$$

$$\begin{aligned} &\text{Inductance parameter} \\ \Lambda &= L_o/L_{do}r_o \end{aligned} \quad (5b)$$

$$\begin{aligned} &\text{Liner and circuit resistance parameters} \\ \theta &= R_o/(L_o/C)^{1/2}, \quad \theta_c = R_c/(L_o/C)^{1/2} \end{aligned} \quad (5c,d)$$

$$\begin{aligned} &\text{Liner heating parameter} \\ K &= \beta\eta_o CV_o^2/[A^2(L_o/C)^{1/2}] \end{aligned} \quad (5e)$$

Specification of these dimensionless parameters provides normalized solutions for the circuit and liner behavior as shown in Figures 2 and 3.

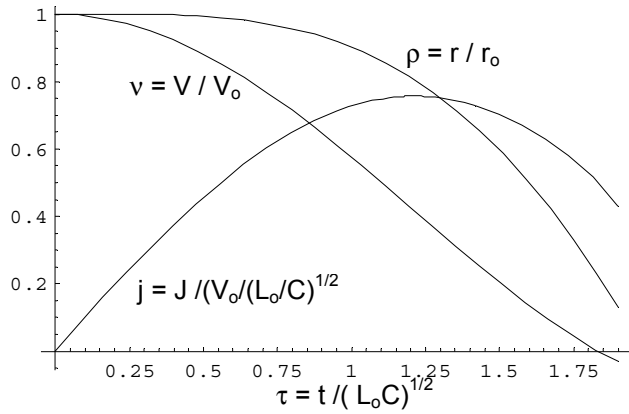


Figure 2. Normalized solution for current, voltage and liner radius, $\Gamma = 0.75$, $\Lambda = 2$, $\theta = 0.001$, $\theta_c = 0.2$, $K = 2$.

III. SCALING RELATIONS

For any set of the dimensionless parameters (Eqns. 5), we can return to dimensional units in terms of the initial radius and length of the liner,

while preserving the normalized liner and circuit behaviors:

$$\begin{aligned} &\text{Initial thickness} \\ \delta_o &= (\eta_o/\mu) [2\beta\rho\Gamma/K\theta]^{1/2} \end{aligned} \quad (6a)$$

For aluminum, $\eta_o = 2.7 \times 10^{-8} \Omega\text{-m}$, $\beta = 2.1 \times 10^{-9} \text{m}^3/\text{J}$, and $\rho = 2.7 \times 10^3 \text{kg/m}^3$, so with $\Gamma = 0.75$, $\theta = 0.001$, and $K = 2$, $\delta_o = 1.4 \text{ mm}$. Note that this is independent of scale size.

$$\begin{aligned} &\text{Inductance} \\ L_o &= (\mu h/2\pi) \Lambda \\ &= 20 \text{ nH}, \text{ for } \Lambda = 2 \text{ and } h = 5 \text{ cm} \end{aligned} \quad (6b)$$

$$\begin{aligned} &\text{Capacitance} \\ C &= 2\beta\rho\theta\Gamma\Lambda r_o^2/(\mu h/2\pi)K \\ &= 2126 \mu\text{f}, \text{ for } r_o = 5 \text{ cm}, h = 5 \text{ cm} \end{aligned} \quad (6c)$$

$$\begin{aligned} &\text{Characteristic impedance} \\ Z_c &= (L_o/C)^{1/2} \\ &= 3.1 \text{ m}\Omega \end{aligned} \quad (6d)$$

$$\begin{aligned} &\text{Characteristic time} \\ t_c &= (L_o C)^{1/2} \\ &= 6.5 \mu\text{s}, \\ &\text{for } \tau = 1.6 \text{ (} r_o/r = 2 \text{), } t = 10.4 \mu\text{s} \end{aligned} \quad (6e)$$

$$\begin{aligned} &\text{Liner speed at above time, with } \omega = 0.4 \\ u &= \omega Z_c / L_{do} \\ &= 6.2 \text{ km/s} \end{aligned} \quad (6f)$$

$$\begin{aligned} &\text{Initial Stored Energy} \\ W_o &= 2\pi r_o h (\eta_o/\mu) [K\rho\Gamma/2\beta]^{1/2}/\theta^{3/2} \\ &= 10.5 \text{ MJ}, \text{ corresponding to } V_o = 99 \text{ kV} \end{aligned} \quad (6g)$$

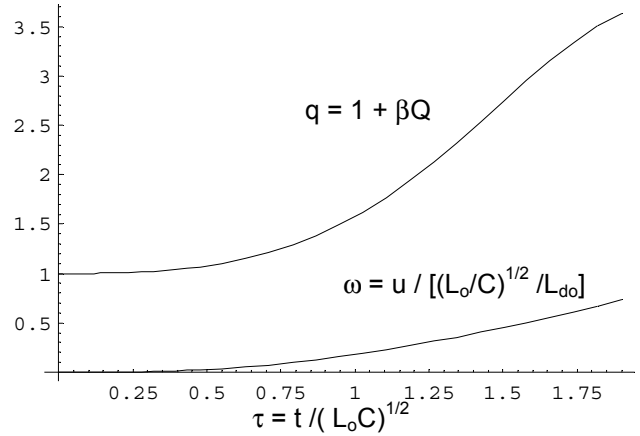


Figure 3. Normalized solution for resistance increase and liner speed, $\Gamma = 0.75$, $\Lambda = 2$, $\theta = 0.001$, $\theta_c = 0.2$, $K = 2$.

If we change the liner dimensions to $r_o = 2 \text{ cm}$ and $h = 1 \text{ cm}$, the various dimensional quantities change (except for the liner thickness). Of particular concern is the decrease of initial circuit inductance to 4 nH, along with a

reduction of stored energy to 0.85 MJ. Competition with minimum values for parasitic inductance of the capacitors and switches thus begins to intrude on the system design. Furthermore, the peak magnetic field on the outer surface of the liner increases from about one megagauss for $r_o = 5$ cm to 1.65 MG for the smaller liner. In the linear diffusion regime, the surface temperature scales as the square of the magnetic field increasing the challenge of controlling the liner material. Both the magnetic field and the electric field scale as $r_o^{-1/2}$ in this fully-diffused model, (and are independent of liner length).

IV. OTHER CIRCUITRY

Transformer techniques can mitigate the effects of decreasing circuit inductance with reduced size by allowing a better match between the primary storage circuit and the relatively low inductance change associated with liner implosion (\sim several nH). A system in which each turn of the transformer primary has an opening-switch has the best electrical performance, but can be cumbersome. A simple current-step-up transformer coupled directly to the liner load (Fig. 4) may be adequate.

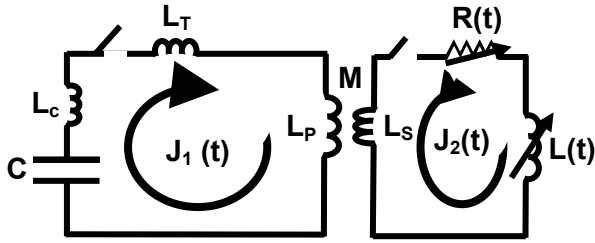


Figure 4. Liner driven by transformer-coupled circuit.

The dimensionless parameters are similar to the direct-drive case, but referenced appropriately to the primary or secondary inductances. We also need additional parameters, such as the usual coupling coefficient, $k = M/(L_p L_s)^{1/2}$, and normalized uncoupled inductances

$$\lambda_1 = L_1/L_p, \quad \lambda_o = L_o/L_s, \quad \lambda(t) = L(t)/L_s \quad (7)$$

These parameters combine as

$$\kappa = k^2 / [(1 + \lambda_1)(1 + \lambda_o) - k^2] (1 + \lambda_o) \quad (8)$$

From the ideal, static circuit ($R = dL/dt = 0$), we have the equivalent inductance

$$L_E = L_p [k/(1 + \lambda_o)]^2 / \kappa \quad (9)$$

The normalized primary and secondary currents are, respectively

$$j_1 = J_1 / [V_o / (L_E/C)^{1/2}], \quad j_2 = J_2 / [\kappa^{1/2} V_o / (L_s/C)^{1/2}] \quad (10)$$

The dimensionless parameters for system behavior now become:

Dynamic parameter

$$\Gamma = \kappa [CV_o^2/2m] / [L_s / L_{do} (L_E/C)^{1/2}]^2 \quad (11a)$$

Inductance parameter

$$\Lambda = L_s / L_{do} r_o \quad (11b)$$

Liner and circuit resistance parameters

$$\theta = (L_E/C)^{1/2} / (L_s/R_o), \quad \theta_c = (L_E/C)^{1/2} / (L_s/R_c) \quad (11c,d)$$

Liner heating parameter

$$K = \kappa (\beta \eta_o / A^2) CV_o^2 (L_E/C)^{1/2} / L_s \quad (11e)$$

Figure 5 displays the normalized solutions for circuit and liner behavior, with $k = 0.9$, and $\lambda_o = \lambda_1 = 0.1$, and for the same values of parameters used in the simple circuit.

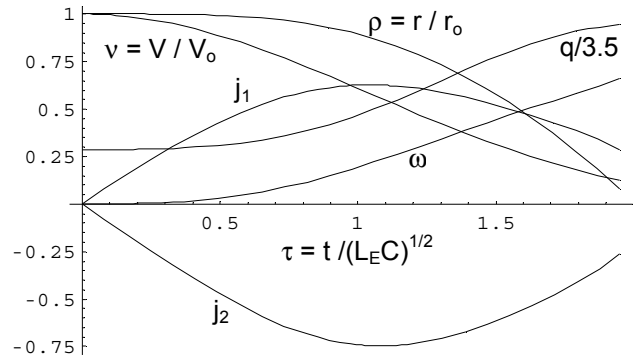


Figure 5. Normalized solutions for transformer-coupled circuit. Resistance-increase q divided by 3.5 to fit graph.

For $r_o = 2$ cm and $h = 1$ cm, and a 12-turn primary, the initial energy is 460 kJ ($C = 32.5$ μ f, $V_o = 167$ kV), $L_s = 4$ nH and $L_p = 580$ nH, allowing $L_1 = 58$ nH, which is a substantial gain over the inductance of the simple circuit, $L_o = 4$ nH.